CS5311 Homework #3 (Spring 2002)

Due: Thursday, March 21, 2002 before class

100 points

1. **[5 points]** What is wrong with the following diagonalization proof?
   - We show that there is a partially computable function that is not in the list Φ_0, Φ_1, Φ_2, ..., Φ_i, ...
   - Let $f(n) = 1 + \Phi(n, n) = 1 + \Phi_n(n)$
   - Then, f is partially computable
   - Since f is not equal to Φ_n for every n, f is not in the list.
2. **[10 points]** Suppose g is a computable function. Show that there exists a computable function f such that for all x, $W_{f(x)} = W_x \cup \{g(x)\}$. (**Hint:** Use the s-m-n Theorem to construct f).
3. **[10 points]** Show that if A is a r.e. set, then $B = \cup_{x \in A} W_x$ is also a r.e. set. (**Hint:** Use the Projection Theorem)
4. **[10 points]** Show that $A = \{ n \mid \{0, 1, \ldots, n\} \subseteq W_n \}$ is r.e. (**Hint:** Use the Projection Theorem)
5. **[10 points]** Assume that A and B are both nonempty *proper* subsets of $\mathcal{N}$. Prove that if $A - B$ and $B - A$ are both recursive, then $A \leq_{\mathrm{m}} B$.
6. **[15 points]** Let FINITE $= \{ x \mid x \in \mathcal{N}$ and W_x is finite $\}$. Show that $\bar{K} \leq_{\mathrm{m}}$ FINITE. What does this reduction mean?
7. **[15 points]** Let TOTAL $= \{ x \mid \Phi_x$ is total $\}$. Show that $\bar{K} \leq_{\mathrm{m}}$ TOTAL.
8. **[15 points]** Let TOTAL $= \{ x \mid \Phi_x$ is total $\}$. Show that TOTAL is not recursive using the Recursion Theorem.
9. **[10 points]** Show that there is an n satisfying $W_n = K$ and $n \in K$. (**Hint:** Use the Recursion Theorem)

Important Notes

You can use a counterexample to disprove a proposition. But, a general argument is required to prove a proposition. I will **not** give any credit to a conceptually incorrect "proof" or a "disproof." Always supply a proof to your claim. Refer to *Notes for Writing and Submitting Homework* for the details.

Print this page and attach it as the last page of your submission

Grade Report

Problem	*Assigned*	*Received*
1	5	
2	10	
3	10	
4	10	
5	10	
6	15	
7	15	
8	15	
9	10	
Total	100	