CS5311 Homework #4 (Spring 2002)

Due: Tuesday, April 16, 2002 **before class**

100 points

1. For each of the following sets, use Rice's Theorem and its extensions to determine if it is recursive, r.e. but not recursive, or not r.e. Prove your claim; otherwise, you will receive **no** credit. Also make sure you cite the known facts properly.
   (a) [**8 points**] $A_1 = \{\ x \mid \text{Range } \Phi_x \text{ is finite }\}$
   (b) [**8 points**] $A_2 = \{\ x \mid W_x = \{\ x\ \}\ \}$
   (c) [**8 points**] $A_3 = \{\ (x, y) \mid \text{for every } a \in \text{Domain}(\Phi_x) \cap \text{Domain}(\Phi_y),\ \Phi_x(a) < \Phi_y(a)\ \}$
   (d) [**8 points**] $A_4 = \{\ x \mid \text{Range}(\Phi_x) = \mathcal{N}\ \}$
   (e) [**8 points**] $A_5 = \{\ x \mid a \notin \text{Range}(\Phi_x)\ \}$ for a given fixed a.

2. [**10 points**] Consider the following proof of **P**$\neq$**NP**.
   (a) We shall prove by contradiction.
   (b) Assume that **P**=**NP**.
   (c) Then, SAT is in **P**, and there is a k such that SAT can be solved in $O(|x|^k)$ time, where x is an input to SAT.
   (d) Since SAT is **NP**-complete, every problem in **NP** can be reduced to SAT in polynomial time and be solved in $O(|x|^k)$. Thus, we conclude that **NP** is a subset of **P**.
   (e) However, since we know that there are problems in **P** that requires a time complexity of at least $O(|x|^{k+1})$, we have a contradiction. Therefore, the assumption is incorrect, and **P**$\neq$**NP**.

   What is wrong with this proof? Clearly identify the problem(s) and elaborate your findings.

3. [**12 points**] Given two undirected graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, G_1 is *isomorphic* to G_2, denoted as $G_1 \cong G_2$, if there is a one-to-one function $f : V_1 \to V_2$ such that for all u, $v \in V_1$, there is an edge between u and v if and only if there is an edge between $f(u)$ and $f(v)$. Consider the following problem:

Problem: Given two undirected graphs $G = (V, E)$ and $H = (W, F)$, is there a subgraph of $H = (W, F)$ that is isomorphic to G?

   Show that this problem is **NP**-complete.

4. [**10 points**] Is 2SAT in CNF in **P** or **NP**-complete? Make your claim first and prove your claim. If you believe 2SAT is **NP**-complete, show the reduction and prove its correctness. If you believe 2SAT is in **P**, design an algorithm to solve it, prove its correctness and analyze its time complexity.

5. [**12 points**] Consider the following problem:

   > Given a family of sets S_1, S_2, ..., S_n, is there a subfamily of k sets S_{i_1}, S_{i_2}, ..., S_{i_k} such that the following holds?
   >
   > $$\cup_{j=1}^{k} S_{i_j} = \cup_{j=1}^{n} S_j$$
   >
   > Note that all sets are finite sets whose members may not be integers.

   Show that this problem is **NP**-complete by reducing the vertex cover problem to it.

6. A set $A \subseteq V$ of a graph $G = (V, E)$ is an *independent set* of size k if and only if $|A| = k$ and for any two vertices u and v in A we have $\overline{uv} \notin E$. The *independent set of size k problem*, IS_k, is

   > Given a graph $G = (V, E)$ and a positive integer k, determine whether G has an independent set of size k.

   The *n-queens problem* nQ is the following:

   > Place n queens in an $n \times n$ chessboard without them attacking each other, where $n \geq 4$.

   Note that a queen on row s and column t can attack all positions on row s, column t, and the two diagonals containing position (s, t).

   Do the following problems:

   (a) [**10 points**] Show that $n\mathrm{Q} \leq_{\mathrm{p}} \mathrm{IS}_k$.

   (b) [**6 points**] B.-J. Falkowski and L. Schmitz in 1986[1] and M. Reichling in 1987[2] published a set of formalæ that provides *one* solution to the nQ problem. Does this fact imply that the independent set problem is in **P** rather than **NP**-complete? Elaborate your findings. Providing only an answer receives **no** credit.

[1] B.-J. Falkowski and L. Schmitz, A Note on the Queens' Problem, *Information Processing Letters*, Vol. 23 (1986), pp. 39–46.

[2] M. Reichling, A Simplified Solution of the N Queens' Problem, *Information Processing Letters*, Vol. 25 (1987), pp. 253–255.

Print this page and attach it as the last page of your submission

Grade Report

Problem		*Assigned*	*Received*
1	a	8	
	b	8	
	c	8	
	d	8	
	e	8	
2		10	
3		12	
4		10	
5		12	
6	a	10	
	b	6	
Total		100	