CS5311 Homework #5 (Spring 2002)

Due: Thursday, May 2, 2002

100 points

1. This problem is about the Reachability problem: Given a directed graph $G = (V, E)$ and two vertices a and b, is there a directed path from a to b?
   (a) [**5 points**] Define **NL**-complete using log-space reduction.
   (b) [**20 points**] Show that Reachability is in **NL**.
   (c) [**20 points**] Show that Reachability is **NL**-complete.

2. [**10 points**] Show that if a **P**-complete problem can be solved in log-space, then all problems in **P** can be solved in log-space.

3. [**10 points**] Show that if **PSPACE-complete** $\cap$ **NP-complete** $\neq \emptyset$ holds, then we have **PSPACE** $=$ **NP** and **PSPACE-complete** $=$ **NP-complete**. Here, **PSPACE**, **PSPACE-complete**, **NP** and **NP-complete** are the classes of **PSPACE**, **PSPACE**-complete, **NP** and **NP**-complete languages.

4. [**15 points**] In the definition of **P**-complete, we use log-space reduction, which has a polynomial time bound. Now define **linearP**-complete as follows:

   > A language $L \in$ **P** is **linearP**-complete if and only if every language Q in **P** can be reduced to L with a linear time bound (*i.e.*, $O(|x|)$, where $x \in Q$ is an input), denoted as $Q \leq_{\text{linear}} L$.

   How large/small is this **linearP**-complete class? Since a linear time bound is less than a polynomial time bound, it is obvious that **linearP**-complete is a proper subset of **P**-complete. Could this set be too small?

5. [**20 points**] We proved in class that the CIRCUIT VALUE problem is **P**-complete. Show that the CIRCUIT VALUE problem remains **P**-complete even if the circuit does not contain the negation operator $\neg$.

Print this page and attach it as the last page of your submission

Grade Report

Problem		*Possible*	*Received*
1	a	5	
	b	20	
	c	20	
2		10	
3		10	
4		15	
5		20	
Total		100	